Announcements

1) HW 5 up - Webwork and supplement, due Friday after break

Back to shifts

In $\mathbb{R}^2$,

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} x+h \\ y \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} x \\ y+k \end{bmatrix}$$

We saw this wasn't given by a matrix.

Fix: Homogeneous coordinates.

Put $\begin{bmatrix} x \\ y \end{bmatrix}$ into $\mathbb{R}^3$ as

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Then to shift horizontally by h, we apply the matrix

$$\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ to } \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Similarly, for shifting vertically by k, apply the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \text{ to } \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Example!

The shift

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} x+56 \\ y-7 \end{bmatrix}$$

is given by what matrix in $\mathbb{R}^3$ acting on homogeneous coordinates?

$$\boxed{A = \begin{bmatrix} 1 & 0 & 56 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix}}$$

Check!

$$\begin{bmatrix} 1 & 0 & 56 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x+56 \\ y-7 \\ 1 \end{bmatrix} \checkmark$$

Scaling:

Contract or expand an image by a given factor $c > 0$. Apply the matrix

$$\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} = c I_2$$

to $\begin{bmatrix} x \\ y \end{bmatrix}$.

Example 2:

Find the matrix that scales all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ up by a factor 56.

$$A = \begin{bmatrix} 56 & 0 \\ 0 & 56 \end{bmatrix}$$

If we said scale down by a factor of 56, we'd use

$$B = \begin{bmatrix} \frac{1}{56} & 0 \\ 0 & \frac{1}{56} \end{bmatrix}.$$

Note: (inclusion into $M_3(\mathbb{R})$)

Include rotations and scalings into $M_3(\mathbb{R})$ by the formula

$$A \mapsto \begin{bmatrix} A & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

for A in $M_2(\mathbb{R})$

Mathematica Example

Under "Resources" on CTools. Feel free to toy with it!

Chapter 4

Real Vector Spaces

The idea: a vector space should have elements (vectors) that you can add and multiply by real numbers.

Definition: (Vector Space)

A vector space is a set V with two operations "$+$" and "$\cdot$",

$$+ : (V, V) \to V$$

$$\cdot : (\mathbb{R}, V) \to V$$

Such that

Satisfying

1) Associativity of "+":

If $v, w,$ and u are vectors in V,

$$(v+w)+u = v+(w+u)$$

2) Identity of "+":

There is a vector 0_V in V with $0_V + w = w + 0_V = w$ for all w in V

3) Inverses for "+"

For all w in V, there is a u in V with

$$\boxed{w + u = u + w = 0_V}$$

Usually write $u = -w$.

4) Commutativity of "+"

For all u, w in V,

$$\boxed{u + w = w + u}$$

5) Associativity of "$\cdot$"

For all real numbers a, b and w in V,

$$\underbrace{a \cdot (b \cdot}_{\text{vector space mult.}} v) = (a \underset{\text{real number mult}}{\cdot} b) \overset{\text{vector space mult}}{\cdot} v$$

6) Identity for "$\cdot$"

For all w in V,

$$1 \cdot w = w$$

7) Distributivity of "$\cdot$" over "$+$":

For all a, b in $\mathbb{R}$ and v, w in V,

$$\boxed{\begin{aligned} (a+b)\cdot w &= a\cdot w + b\cdot w \\ a\cdot(w+v) &= a\cdot w + a\cdot v \end{aligned}}$$

Example 3: $\mathbb{R}^n$ is a vector space.

"$+$" is component-wise addition (usual vector addition).

"$\cdot$" is multiplying each component (usual scalar multiplication)

Example 4: $M_n(\mathbb{R})$ is a vector space. (vectors are matrices)

"$+$" is matrix addition

"$\cdot$" is multiplication of each component.

In fact, $M_n(\mathbb{R})$ is just $\mathbb{R}^{n^2}$ as a vector space.

Identity for $M_n(\mathbb{R})$:

the zero matrix.

Example 5: $C(\mathbb{R})$

means continuous functions from $\mathbb{R}$ to $\mathbb{R}$. The vectors are the functions.

Given two functions f, g in $C(\mathbb{R})$, we define

$f+g$ by

$$\boxed{(f+g)(x) = f(x)+g(x)}$$

for all x in $\mathbb{R}$

If c is a real number, define $c \cdot f$ by

$$\boxed{(c \cdot f)(x) = c \cdot f(x)}$$

for all x in $\mathbb{R}$.

The identity is the function that is constantly zero.